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Insurers generally rely upon certain tools to manage the asset risks associated with living benefits features so popular in variable annuity contracts, including limitations on the election of investment options, and limits on withdrawals in excess of those provided under these benefits, reductions to guaranteed death benefits proportional to amounts withdrawn, investment hedges in their general accounts to offset investment losses with ongoing rebalancing relative to product portfolio risks, and computer programs that provide sophisticated stochastic analysis of the asset portfolio. While applying significant resources to measuring and managing asset risk, insurers tended to rely upon their traditional deterministic methods to price the mortality component, often using a best estimate with some explicit or implicit margin. However, this article demonstrates that shifts in future mortality improvement rates may produce additional asset shortfalls that are not so readily managed by use of financial hedges.

Some insurers believe that longevity risk is offset by mortality risk from their life insurance blocks. However, changes in mortality for older annuitant ages have historically proven to be poorly correlated with measured changes for younger insurance ages. For example, the correlation of average long-term changes in a 75 year old male mortality rate and a 45 year old male mortality rate is $-63\%^2$. In another example, during the 1918 flue pandemic, mortality rates increased at insurance ages (i.e., 30s and 40s), but decreased at annuitant ages (i.e., ages 65+).

We propose that insurers should utilize a stochastic methodology for evaluating the risk, allowing pricing to reflect the risk tolerance of the insurer. The following analysis describes both a deterministic and stochastic methodology to establish the value added with a stochastic analysis approach

Deterministic Modeling: The following example demonstrates that there may be significant cost associated with longevity not captured by investment hedging.

Product Design: We start with a very simple design for a variable annuity (VA) with a guaranteed minimum withdrawal benefit (GMWB):

- Male, attained age 60;
- 12/31/2011 Baseline Account Value = 150,000;
- 12/31/2011 Withdrawal Benefit Base = 150,000;
- GMWB Lifetime Annual 5% withdrawals of Withdrawal Benefit Base (\$7,500) starting at Age 60; and
- Death Benefit equals remaining AV (i.e., the contract provides no additional GMDB), assumed to be payable end of year of death.

A 4.98% separate account investment return ("Target Return") was assumed to make the contract self-sustaining. That is, at this Target Return, withdrawals deplete contract value exactly at the end of the table (age 120), and the GMWB generates no additional value regardless of when the insured dies. The following table demonstrates the balance between withdrawals and death benefits:

Distribution of Future Benefits Under Various Mortality Assumptions								
NPV on 12/31/2011 at 4.98% Assumed Future Investment Return								
150% Expected Improvement w/Adjustments for Survival to								
Mortality Assumption	Death on 12/31/2011	Expected Mortality	Specific CODs	Age 120				
NPV Future Death Benefits (assumed payable EOY)	\$150,000	\$39,993	\$36,917	\$0				
NPV Future Withdrawals (assumed payable BOY)	\$0	\$110,007	\$113,083	\$150,000				
Total (NPV Future Benefits)	\$150,000	\$150,000	\$150,000	\$150,000				

Effect of Investment Shortfalls: When the account value is less than the benefit base, or future investment returns are less than the assumed interest rate, there will be a shortfall that must be paid out of company surplus. For this analysis, the value of the shortfall equals any positive excess of the present value of projected future benefits (withdrawals plus death benefits) over the account value.

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² Long-term correlation is measured using the average annual mortality improvement of the general US population for each attained age over the three consecutive 10-year intervals consisting of 1977-1987, 1987-1997, and 1997-2007.

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We looked at a range of scenarios in which future mortality matches expectations but investment returns do not achieve the Target Return: (a) invested assets are less than the Withdrawal Base or (b) future investment returns are less than the Target Return of 4.98%. Using this simplistic example, we have a mechanism by which to set aside the risk associated with investment volatility, and isolate the effect of the volatility of mortality improvement.

Sensitivity to Adverse Investment Scenarios							
(Mortality Assumed at Expected Levels)							
Description	Baseline	(a) Withdrawal Benefit Base Exceeds AV	(b) Future Net Investment Rates Less than Expected				
Assumed Future Net Investment Return	4.98%	4.98%	4.50%	4.00%	3.00%	2.00%	
Assumed Account Value 12/31/2011	\$150,000	\$120,000	\$150,000	\$150,000	\$150,000	\$150,000	
Assumed Withdrawal Benefit Base 12/31/2011	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	
NPV Future Withdrawals at Net Investment Return	\$110,007	\$110,007	\$115,637	\$122,097	\$137,072	\$155,439	
NPV Future Death Benefits at Net Investment Return	\$39,993	\$15,368	\$34,428	\$29,368	\$22,687	\$18,673	
Total NPV of Future Benefits at Net Investment Return	\$150,000	\$125,375	\$150,065	\$151,465	\$159,759	\$174,111	
Shortfall Attributable to Reduced Investment Return*	\$0	\$5,375	\$65	\$1,465	\$9,759	\$24,111	
* PV Future Benefits in Excess of Account Value (Offset by Theoretical Investment Hedge)							

When results reflect both past and future investment returns below expected, the combined shortfall is greater than the sum of the separate shortfall amounts. For the purposes of this analysis, we defined an imperfect Theoretical Investment Hedge that is sufficient to cover the shortfall amounts generated by these asset shortfalls.

Sensitivity to Shortfalls in Past & Future Investment Returns							
(Mortality Assumed at Expected Levels)							
NPV on 12/31/2011 at Future Investment Return							
Assumed Future Investment Return 4.98% 4.50% 4.00% 3.00% 2.00					2.00%		
Reduced Assumed Account Value 12/31/2011	\$120,000	\$120,000	\$120,000	\$120,000	\$120,000		
Total NPV of Future Benefits at Future Investment Return	\$125,375	\$129,475	\$134,686	\$147,837	\$164,916		
Total Shortfall*	\$5,375	\$9,475	\$14,686	\$27,837	\$44,916		
(a) Amount from 20% Shortfall of AV 12/31/2011	\$5,375	\$5,375	\$5,375	\$5,375	\$5,375		
(b) Shortfall Attributable to Reduced Future Return	\$0	\$65	\$1,465	\$9,759	\$24,111		
Implied Additional Shortfall Attributable to Combination	\$ <i>0</i>	\$4,036	\$7,846	\$12,704	\$15,430		

* PV Future Benefits in Excess of Account Value (Offset by Theoretical Investment Hedge)

Assumed Investment Hedge and Mortality Improvement Volatility: If a product is not perfectly immunized against fluctuations in mortality improvement and investment returns, the interactions of these separate volatilities may create significant unforeseen losses. Consider an example where (a) starting Account Value is less than the Withdrawal Benefit Base (i.e., past investment shortfall), (b) future investment returns are less than expected and (c) future mortality improvement is higher than expected. While shortfalls associated with the first two risks would be offset by the Theoretical Investment Hedge, the mortality improvement volatility can exaggerate the risk of loss when assets are not perfectly hedged, leaving shortfalls not offset by the Theoretical Investment Hedge.

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Shortfall in Past & Future Investment Returns and Higher-Than-Expected Mortality Improvement Rates							
NPV on 12/31/2011 at Assumed Future Investment Return							
Assumed Future Investment Return	4.98%	4.50%	4.00%	3.00%	2.00%		
Reduced Assumed Account Value 12/31/2011	\$120,000	\$120,000	\$120,000	\$120,000	\$120,000		
Assumed Future Mortality Improvement	150% Expected With Adjustments for Specific Causes of Death						
Total Shortfall*	\$6,995	\$12,917	\$18,682	\$33,210	\$52,204		
LESS Shortfall Attributable to Past & Future Return**	\$5,375	\$9,475	\$14,686	\$27,837	\$44,916		
Additional Shortfall Attributable to Higher Mort Improvement	\$1,621	\$3,443	\$3,996	\$5,373	\$7,287		
*Excess of PV Future Benefits over Account Value ** Offset by Theoretical Investment Hedge							

Depending on its level of risk tolerance, a company using sensitivity testing to determine the potential surplus costs might decide to hold surplus equal to \$3996, based on the Higher-Than-Expected Mortality Improvement over the theoretically hedged investment risk associated with 4.00% Future Return and Reduced Assumed Account Value of \$120,000.

Note that all of the analysis so far relies upon a deterministic approach that calculates a single value for each set of assumptions without providing any guidance as to the likelihood of that result. To better evaluate the range of potential risks, it is necessary to turn to stochastic liability modeling

Stochastic Modeling: We used REVEAL, a proprietary Milliman software tool³, to calculate the volatility measures of mortality improvement rates from historical US Population data and then generate stochastic projections, starting with the following assumptions:

- Stochastic Projection Around Future Improvement Rates
 - Volatility Developed from Analysis of Historical US Population Mortality Improvement
 - Improvement Over Period 1979-2007, Male and Female, Analyzed in 7-Year Age Groups
 - Historic Long-Term Standard Deviation of Consecutive 7-Year Periods
 - Historic Short-Term (Annual) Standard Deviation
- Volatility Attributable to Shifts by Cause of Death, Consistent with Distribution of Deaths by Attained Age & Gender for US Population (1972-1977):
 - 2% Annual Probability of 25% increase in deaths by Infectious Disease
 - 2% Annual Probability of 25% decrease in deaths by Neoplasm
 - 2% Annual Probability of 25% decrease in deaths by Circulatory Disease

Consistent with the first table, we find that if the future investment returns always achieve the 4.98% target rate, the results are insensitive to mortality results, and the distribution of benefits paid shifts to withdrawals from death benefits as longevity increases. The scenarios were ranked by NPV Future Withdrawals, giving the percentiles and Contingent Tail Expectation (CTE) at various levels. Thus, when the Assumed Investment Return is satisfied, the NPV of future withdrawals and death benefits are in balance, with no shortfall.

³ REVEAL (which stands for Risk and Economic Volatility Evaluation of Annuitant Longevity) is a system developed to analyze longevity risk. REVEAL generates stochastic projections on pension and annuity liabilities with volatile assumptions (i.e., baseline mortality, mortality improvement, extreme mortality and longevity events, and plan participant behavior - such as retirement dates and benefit elections). For more information about REVEAL, please see http://www.milliman.com/expertise/life-financial/products-tools/reveal/.

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CTE Distribution - Assumed Investment Return (4.98%) and Various Mortality Improvement Scenarios								
	Expected tPx (100% 2000 US Annuity w/ 100% Scale G)	Stochastic tPx (Mean)	Stochastic tPx (50% CTE)	Stochastic tPx (70% CTE)	Stochastic tPx (90% CTE)	Stochastic tPx (95% CTE)	Stochastic tPx (99% CTE)	
Account Value 12/31/2011	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	
NPV on 12/31/2011 at 4.98%								
NPV Future Wdrls BOY	\$110,007	\$110,999	\$112,976	\$113,919	\$115,458	\$116,211	\$117,532	
Future DBs EOY	\$39,993	\$39,001	\$37,024	\$36,081	\$34,542	\$33,789	\$32,468	
Total	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	\$150,000	

However, if we consider the mortality improvement volatility where the past and future investment returns are less than the target, consistent with what we saw in the deterministic analysis, the Theoretical Investment Hedge is not sufficient to cover the entire shortfall.

Shortfall in Past and Future Investment Returns and Stochastic Mortality Improvement Rates							
NPV on 12/31/2011 at Assumed Rate of Return							
Assumed Future Return	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	
Assumed AV 12/31/2011	\$120,000	\$120,000	\$120,000	\$ 120,000	\$ 120,000	\$ 120,000	
Selected Stochastic Scenario	Expected tPx	Stochastic tPx (50% CTE)	Stochastic tPx (70% CTE)	Stochastic tPx (90% CTE)	Stochastic tPx (95% CTE)	Stochastic tPx (99% CTE)	
Total Shortfall	\$14,686	\$16,052	\$16,794	\$18,009	\$18,585	\$19,537	
LESS Attributable to Past & Future Return*	\$14,686	\$14,686	\$14,686	\$14,686	\$14,686	\$14,686	
Additional Shortfall Attributable to Improvement	\$0	\$1,366	\$2,108	\$3,323	\$3,900	\$4,851	
* Offset by Theoretical Investment Hedge							

One lesson to be drawn from this analysis is that, by looking at the results of stochastic projection, the actuary may be able to manage the product to a target risk level. In the above example, the company may use a mortality hedge to offset the longevity risk to a desired risk tolerance (e.g., at the 90% CTE equal to \$3,323). Otherwise it may be prudent to hold economic capital for this mortality improvement volatility.

It is worth highlighting the fact that the \$3,323 required for the 90% CTE is \$673 or 17% lower than the \$3,996 that the company may have been utilized based solely on deterministic sensitivity testing. Hence the application of stochastic analysis to the longevity risk has a demonstrable value in illustrating lower levels of economic capital.

Conclusion: Longevity products are growing in importance to the insurance industry and to the aging population as a whole. The use of a target CTE applied to a stochastic projection may turn out to be more or less than the margin for contingencies built into the pricing. (The analysis in this article made no such provision for adverse deviation.) Therefore, the company attempting to protect against longevity risk may be inadvertently holding reserves and capital based on deterministic margins that may be overly conservative relative to historical levels of volatility. This paper introduced some thoughts that are only the first step in an increasingly complex situation.

The types and amounts associated with longevity risk will be very different given elements that, for simplicity, were not considered herein such as alternative GMWB designs, requirements for Statutory and GAAP reserves and policyholder behavior. The demonstrated effects on these items from mortality volatility are waiting to be analyzed in greater depth.